RT-GSFit
(Real Time – Grad-Shafranov Fitter)

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Introduction

Context

- Enhancing position and shape control is a major objective for the ST40 2027 campaign.
- Improved control allows precise management of the gap between plasma-facing components and the Last Closed Flux Surface (LCFS).
- This supports performance optimisation e.g., keeping the plasma close to the central solenoid while remaining in a diverted configuration.
- Importantly, better shape control enables accurate strike-point positioning and ensures diagnostics (e.g., the IR Camera) maintain full visibility.

Aims

- Develop RT-GSFit, a real-time Grad–Shafranov solver (1 ms cadence) to estimate the magnetic field and LCFS.
- Achieve a 1 ms solver cadence, shorter timesteps offer little practical benefit because open loop response times are >10ms.
- Write the code in C to seamlessly integrate into the existing plasma control system (which uses MathWorks Simulink).

Iteration Loop

1. Initial Guess/Calculated $\psi_N(R,Z)$ + Measurements → 2. Calculate Least Squares Solution to get $j_\phi(R,z)$ → 3. Solve Poisson Equation to get $\psi(R,Z)$ → 4. Find magnetic axis and LCFS to get $\psi_N(R,Z)$

1. Input $\psi_N(R,Z)$

- Supply an initial guess for the normalised poloidal flux $\psi_N(R,Z)$ for the first iteration, and then use the ψ_N computed in Stage 4 for all subsequent iterations.
- Provide the latest measurements from Mirnov coils, Rogowski coils, flux loops, and the PF-coil currents supplied by the power-supply units.

2. Calculate Least Squares Solution to get $j_\phi(R,z)$

- We express the toroidal plasma current in the form:

$$j_{\phi,\mathrm{plasma}}(R,z) = \underbrace{a_1(1-\psi_N)R}_{\text{p' term}} + \underbrace{a_2\frac{1-\psi_N}{R}}_{\text{ff' term}} + \boxed{a_3\frac{\partial\psi_N}{\partial z}} \leftarrow \text{Added for numerical stability}$$

where $a_1, a_2, \ldots$ are free parameters.
- We expand the toroidal currents in the metal using 15 eigenfunctions for the inner vacuum chamber (IVC), 1 for the single outer vacuum chamber (OVC), 1 per passive stabilisation ring (4 total), and 1 per case structure (2 total). Thus

$$j_{\phi,\mathrm{metal}}(R,z) = \sum_{i=4}^{25} a_i\, f_i(R,z)$$

- We use Green's functions, $G_i(R,z,R',z')$ to relate the toroidal currents to the i^{th} measurement. This gives the linear system:

$$A\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{25} \end{pmatrix} = \begin{pmatrix} w_1 m_1 \\ w_2 m_2 \\ \vdots \\ w_{56} m_{56} \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

← 56 measurement rows (26 flux loops, 21 Mirnov coils and 9 Rogowski coils).

← 16 regularisation/fake sensor rows of the form $w_i a_i = 0$ for the IVC and OVC degrees of freedom

$$A = \begin{pmatrix}
w_1 \iint (1-\psi_N)R'\, G_1\, dR'dz' & w_1 \iint \frac{1-\psi_N}{R'} G_1\, dR'dz' & \cdots & \cdots & \cdots & \cdots & w_1 \iint f_{25}\, G_1\, dR'dz' \\
w_2 \iint (1-\psi_N)R'\, G_2\, dR'dz' & w_2 \iint \frac{1-\psi_N}{R'} G_2\, dR'dz' & \cdots & \cdots & \cdots & \cdots & w_2 \iint f_{25}\, G_2\, dR'dz' \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
w_{56} \iint (1-\psi_N)R'\, G_{56}\, dR'dz' & w_{56} \iint \frac{1-\psi_N}{R'} G_{56}\, dR'dz' & \cdots & \cdots & \cdots & \cdots & w_{56} \iint f_{25}\, G_{56}\, dR'dz' \\
0 & \cdots & 0 & w_{57} & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & w_{58} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & 0 & \cdots & w_{72}
\end{pmatrix}$$

9 Columns | 16 Columns

- $(w_1, w_2, \ldots, w_{72})$ is the weight vector.
- $(m_1, m_2, \ldots, m_{56})$ is the measurement vector pre-processed to remove PF-coil contributions.
- We take the pseudoinverse of A to find the best-fit coefficients a_i in the least-squares sense and use these to compute the toroidal current distribution $j_\phi(R,z)$.

3. Solve Poisson Equation to get $\psi(R,z)$

- At this stage, $j_\phi(R,z)$ is fixed and we solve:

$$\frac{\partial^2\psi}{\partial R^2} - \frac{1}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial z^2} = -2\pi\mu_0 R j_\phi(R,z)$$

to get $\psi(R,z)$.
- Discretise over a 65×33 finite-difference grid.
- Invert corresponding matrix using Gaussian elimination.
- Use Von Hagenow's method (see e.g. Chap 4.6.4 of [1]) to compute the boundary values.

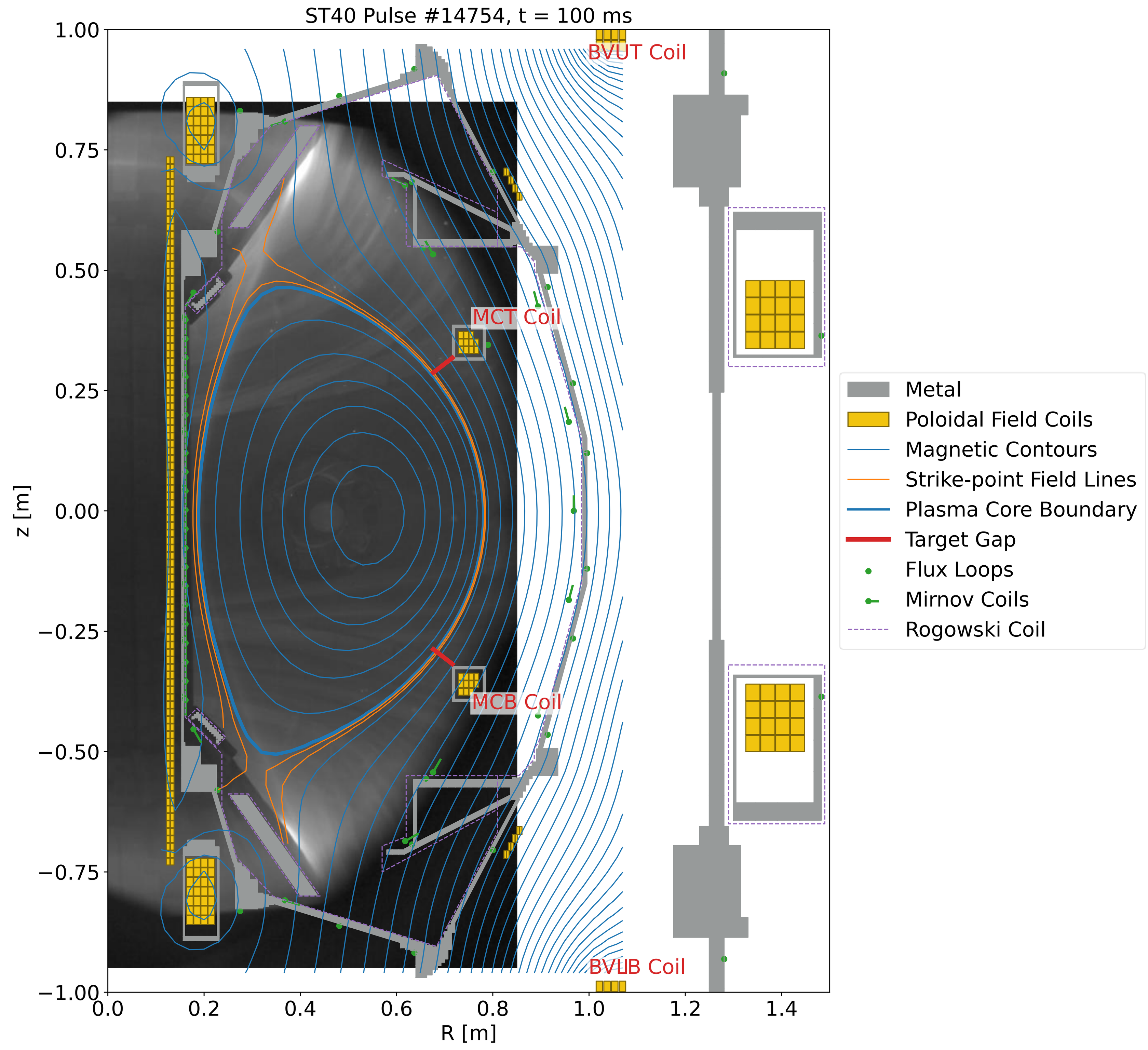
4. Find magnetic axis and LCFS to get $\psi_N(R,Z)$

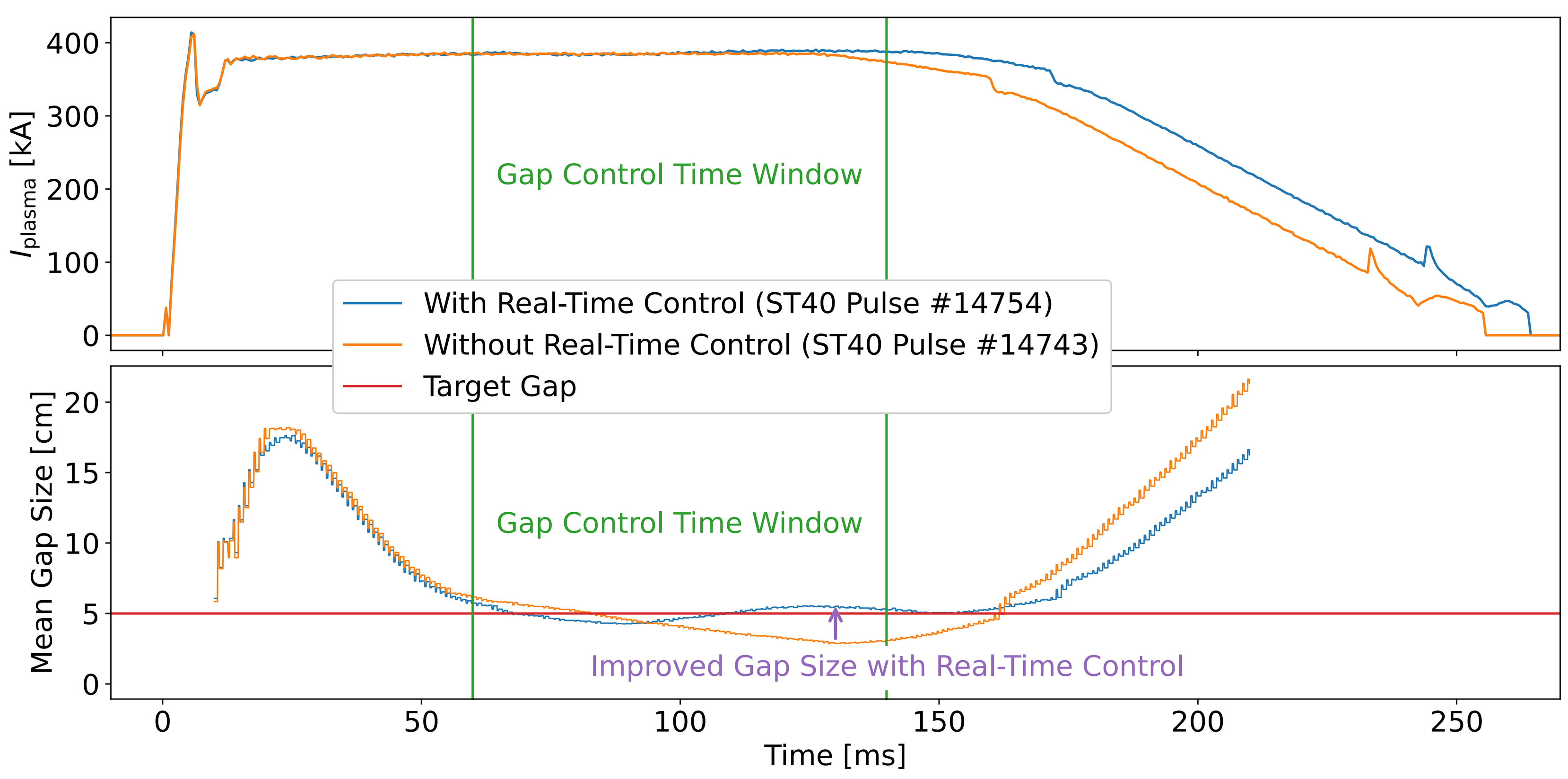
- Use 9-point interpolation/Taylor series:

$$\psi(R,z) = \psi_{i,j} + a\frac{R-R_j}{\Delta R} + b\frac{z-z_i}{\Delta z} + \frac{c}{2}\left(\frac{R-R_j}{\Delta R}\right)^2 + \frac{d}{2}\left(\frac{z-z_i}{\Delta z}\right)^2 + e\frac{(R-R_j)}{\Delta R}\frac{(z-z_i)}{\Delta z},$$

- a, b, c, d, e are centred finite difference approximations of the corresponding derivatives in the Taylor series
- ΔR, Δz are the cell widths and heights.
- Identify stationary points by solving $\partial\psi/\partial R = \partial\psi/\partial z = 0$
- Evaluate the Hessian at each stationary point to classify them as X-points or O-points.
- The magnetic axis value, ψ_A, is set to the O-point with the largest value of ψ.
- The LCFS value, ψ_{LCFS}, is set to the largest ψ value at either the plasma-facing components or any identified X-points.
- Finally calculate ψ_N and ensure $\psi_N = 1$ outside the plasma core using a flood-fill algorithm.

Results





- We integrated RT-GSFit into the ST40 plasma control system during the final weeks of the 2025 campaign, just ahead of the November shutdown.
- The plasma control system now operates with two parallel threads. A fast thread running at 50 µs for vertical stabilisation and total plasma current control. A slower 1 ms for horizontal control.
- The objective was to keep a 5 cm gap between the LCFS and the top/bottom merging compression coils (MCT/MCB) within a defined time window (60–140 ms).
- We used the BVU coils as actuators to control the gaps.
- The results shown are from a pulse using RT-GSFit operating in real-time feedback mode (Pulse #14754), compared with a reference feedforward pulse (Pulse #14743).

Discussion and Future Work

- The results above show that RT-GSFit can already improve horizontal control. The question now if can we use it to control the strike-point location.
- We have never attempted to control the X-point position in ST40 before. This is challenging because small errors in our calculation of ψ can produce large shifts in the inferred X-point location, so high accuracy is essential.
- To improve reconstruction accuracy, we plan to:
  1. Calibrate and incorporate the ST40 diamagnetic flux loop as an additional measurement constraint.
  2. Integrate our real-time Thomson scattering (developed in collaboration with PPPL) data to refine the toroidal plasma current density profile $j_{\phi,\mathrm{plasma}}(R,z)$.
  3. Adopt a more advanced Poisson solver, replacing simple Gaussian elimination with a method such as Buneman [2]. This could allow us to use a higher resolution while keeping the iterations under 1 ms.
- We cannot directly measure currents in the IVC or OVC since we do not have Rogowski coils there. Instead, we rely on regularisation to suppress unphysical vessel currents. Perhaps we should take a circuit equation approach using flux-loop voltage measurements to allow us to directly constrain vessel currents.
- Validate RT-GSFit and control in forward modelling codes such as SOPHIA [3].

References and Acknowledgments

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- Special thanks to the authors of [4] we found it very helpful when developing this code ☺.

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