Mode coupling at the transition region and the validity of line-tied boundary conditions

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Aims

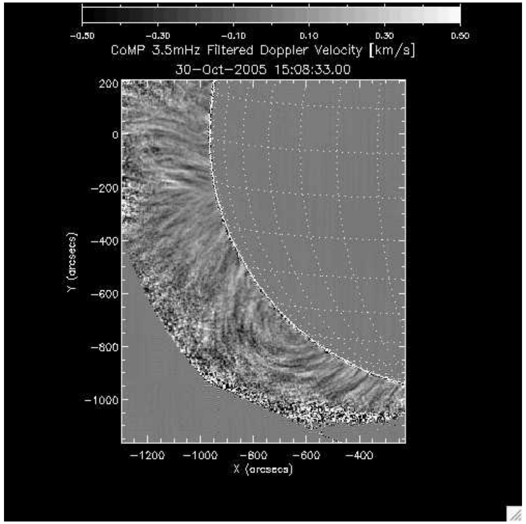
- Show why Fast / Alfvén waves couple at the TR
- Show that polarisation of the waves changes upon reflection
- Test the validity of line-tied BCs

Structure

- **Background**
- Model 1:
  - Line-tied, pulse
- Model 2:
  - Line-tied, normal mode
- Model 3:
  - Chromosphere, normal mode
- Summary and conclusions

Why study MHD waves?

- Ubiquitous
- Coronal heating
- Coronal seismology

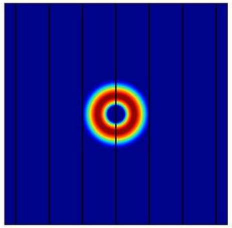


Tomczyk et al. (2007)

Fast vs. Alfvén waves

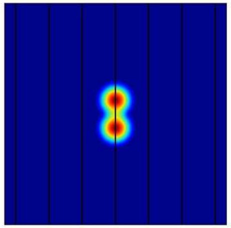
Fast waves:

- Propagate isotropically
- $\frac{\omega}{v_A} = \pm\sqrt{k_x^2 + k_y^2 + k_z^2}$



Alfvén waves:

- Propagate parallel to $\mathbf{B}_0$
- $\frac{\omega}{v_A} = \pm k_\parallel$



Mode conversion

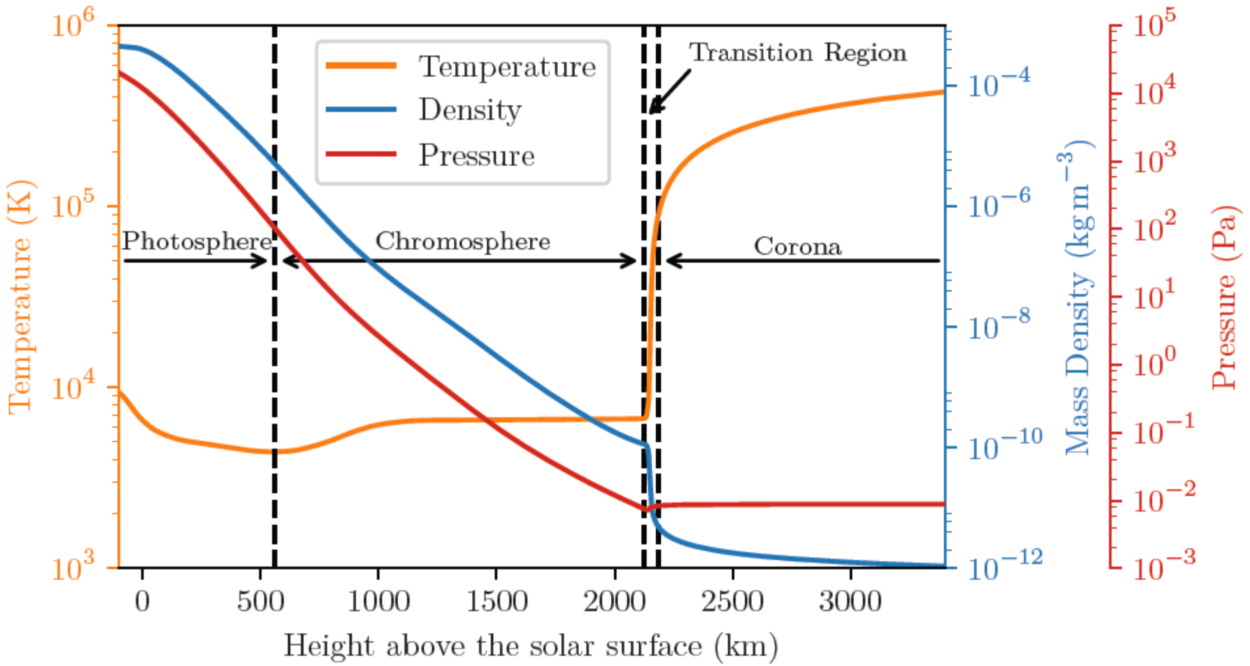
Can occur via:

- Non-linear effects (Verwichte et al., 1999)
- Transition from $\beta < 1$ to $\beta > 1$ plasma (McLaughlin & Hood, 2006)
- Gradients in $v_A \rightarrow$ resonant absorption (Ionson, 1982)

Mode conversion at the TR

- Studied analytically in Halberstadt & Goedbloed (1993, 1994, 1995)
- Numerical approach used in Arregui et al. (2003)
- Cally & Hansen (2011, 2012) suggest that mode conversion from fast waves to Alfvén waves at the transition region enables sufficient energy flux to enter the corona

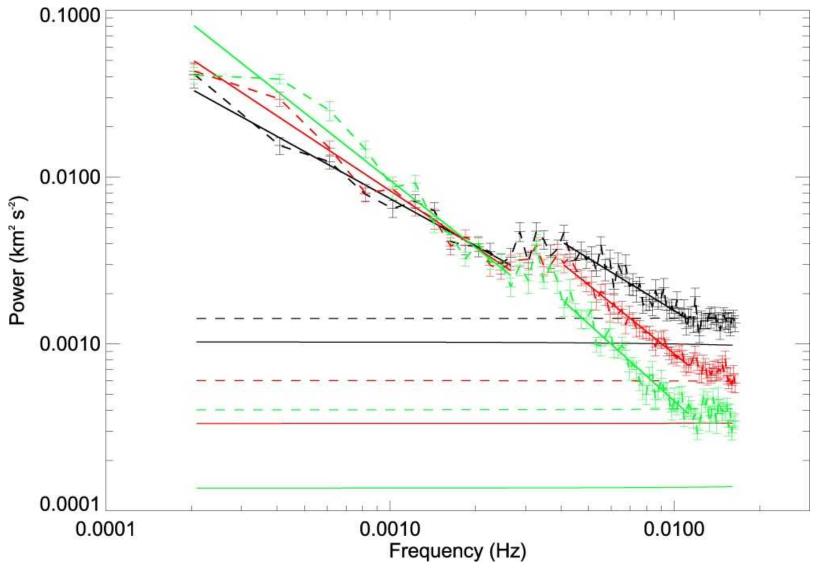
Line-tied (***u***=0) boundary conditions



Vernazza et al. (1981) and Williams (2018)

Normal mode

- $f(\mathbf{r},t) = f_0(\mathbf{r})\exp(i\omega t)$



Morton et al. (2016)

Model and Equations

- Background quantities:

$$\rho = \rho_0 \qquad \boldsymbol{B_0} = B_0 \hat{\boldsymbol{B}}_\mathbf{0}$$

- Perturbations:

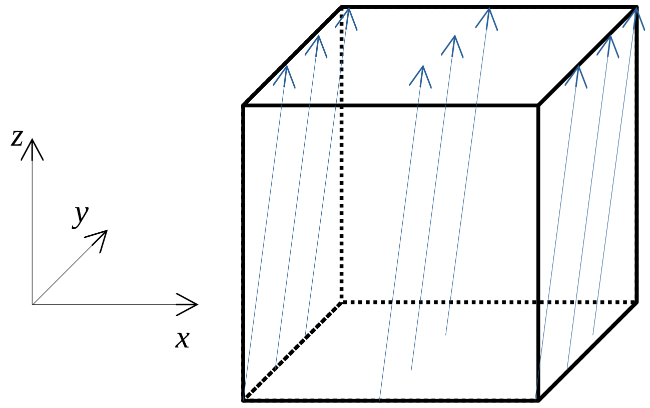
$$\boldsymbol{u} = u_x \hat{\boldsymbol{x}} + u_\perp \hat{\perp} \qquad \boldsymbol{b} = b_x \hat{\boldsymbol{x}} + b_\perp \hat{\perp} + b_\parallel \hat{\boldsymbol{B}}_\mathbf{0}$$

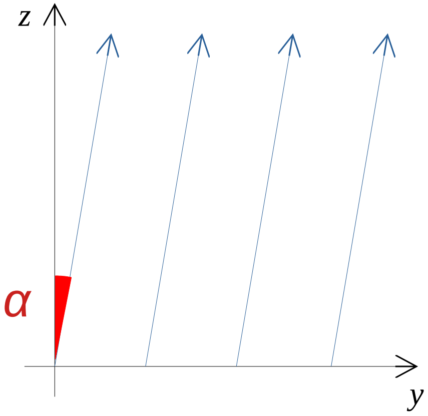
- Unit vectors:

$$\hat{\perp} = \cos(\alpha)\hat{\boldsymbol{y}} - \sin(\alpha)\hat{\boldsymbol{z}} \qquad \hat{\boldsymbol{B}}_\mathbf{0} = \sin(\alpha)\hat{\boldsymbol{y}} + \cos(\alpha)\hat{\boldsymbol{z}}$$

- Equations:

$$\rho_0 \frac{\partial \boldsymbol{u}}{\partial t} = \boldsymbol{j} \times \boldsymbol{B_0} \qquad \frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B_0})$$





Structure

- Background
- **Model 1:**
  - **Line-tied, pulse**
- Model 2:
  - Line-tied, normal mode
- Model 3:
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Numerical scheme

- Leapfrog algorithm
- Based on Zalesak (1979)
- Finite-difference
- Staggered grid
- Second-order accurate

Initial / boundary conditions

- Assume that

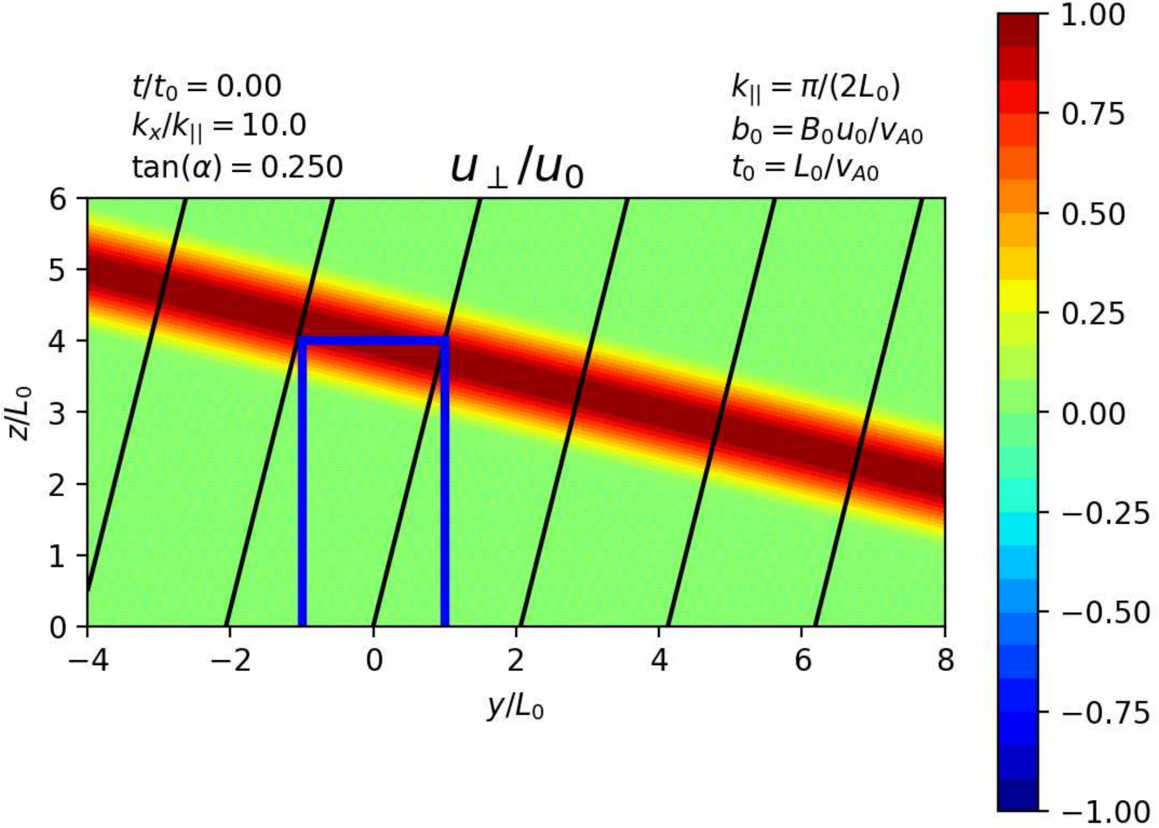
$$u_x, b_x \propto \sin(k_x x)$$

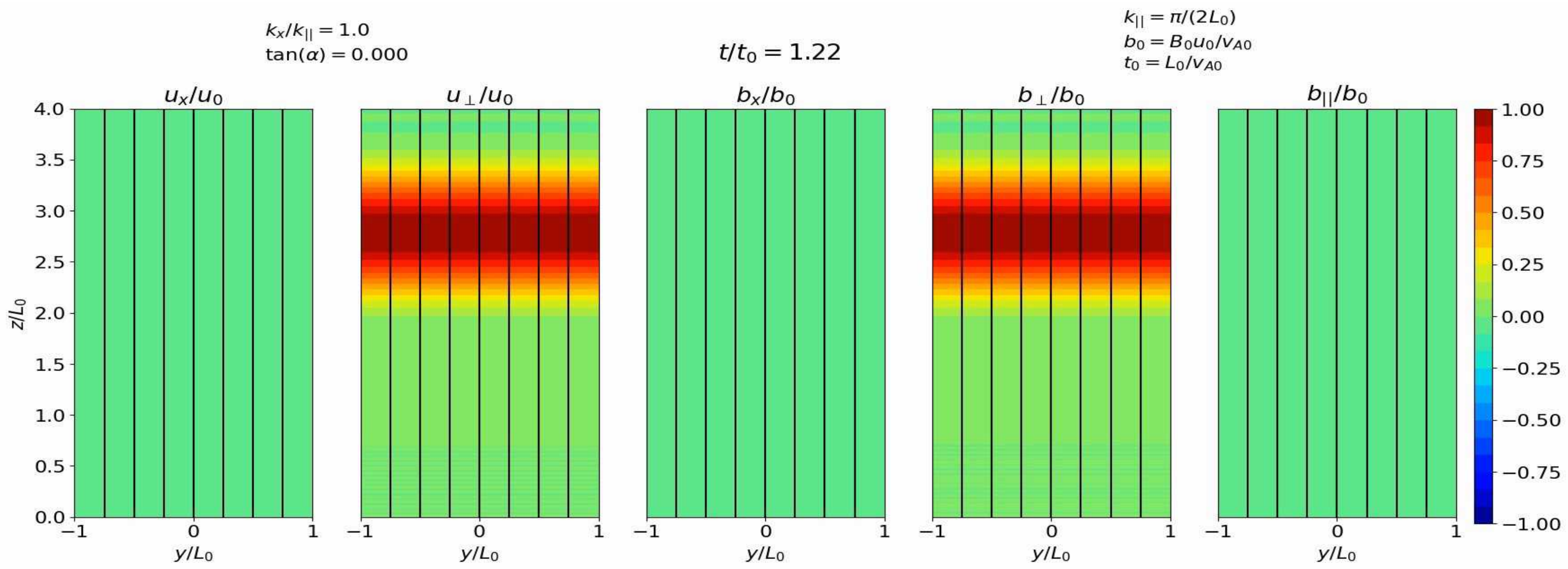
$$u_\perp, u_\perp, b_\parallel \propto \cos(k_x x)$$

- Initial conditions:

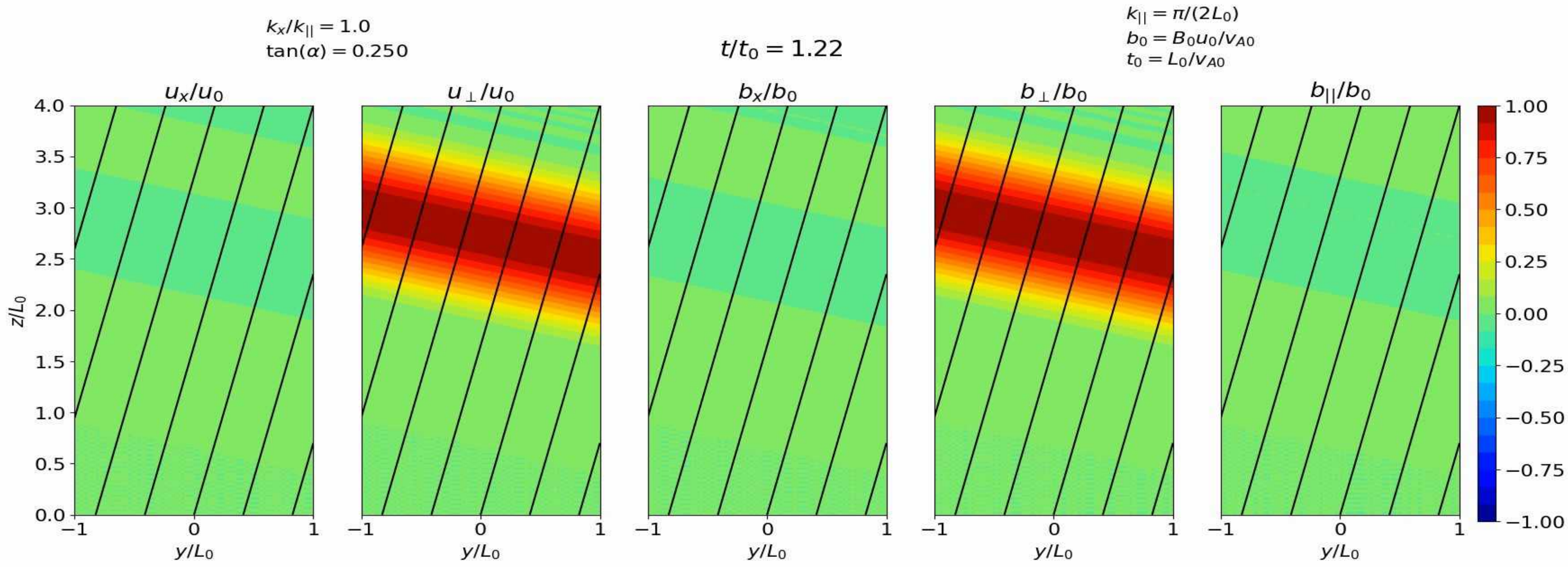
$$\frac{u_\perp}{u_0} = \frac{b_\perp}{b_0} = \begin{cases} \cos^2\theta & \text{if} \quad |\theta| \le \pi/2 \\ 0 & \text{if} \quad |\theta| > \pi/2 \end{cases}$$

$$\theta = k_\parallel \left(y \sin\alpha + (z + 4L_0)\cos\alpha + v_A t \right)$$

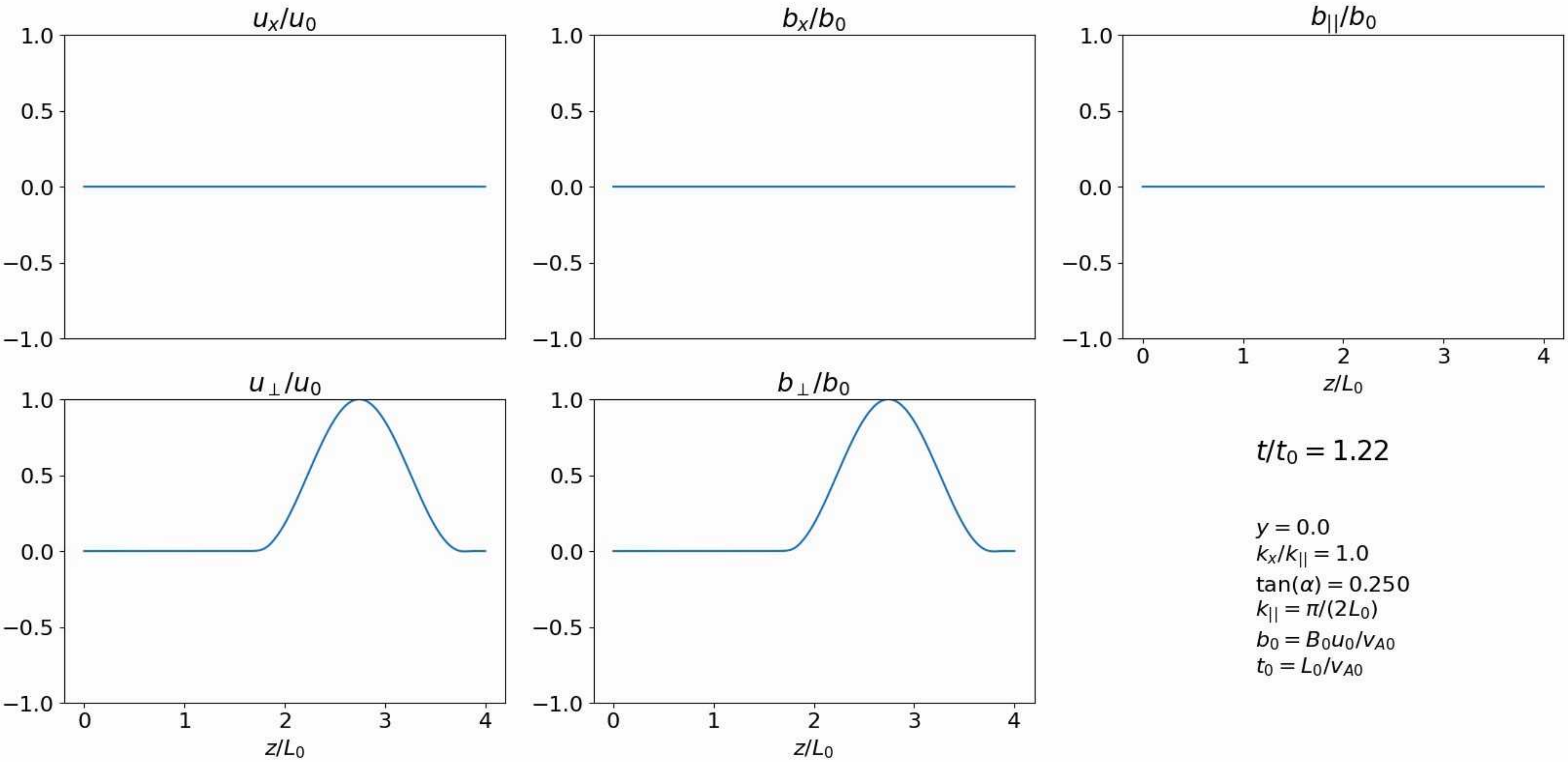




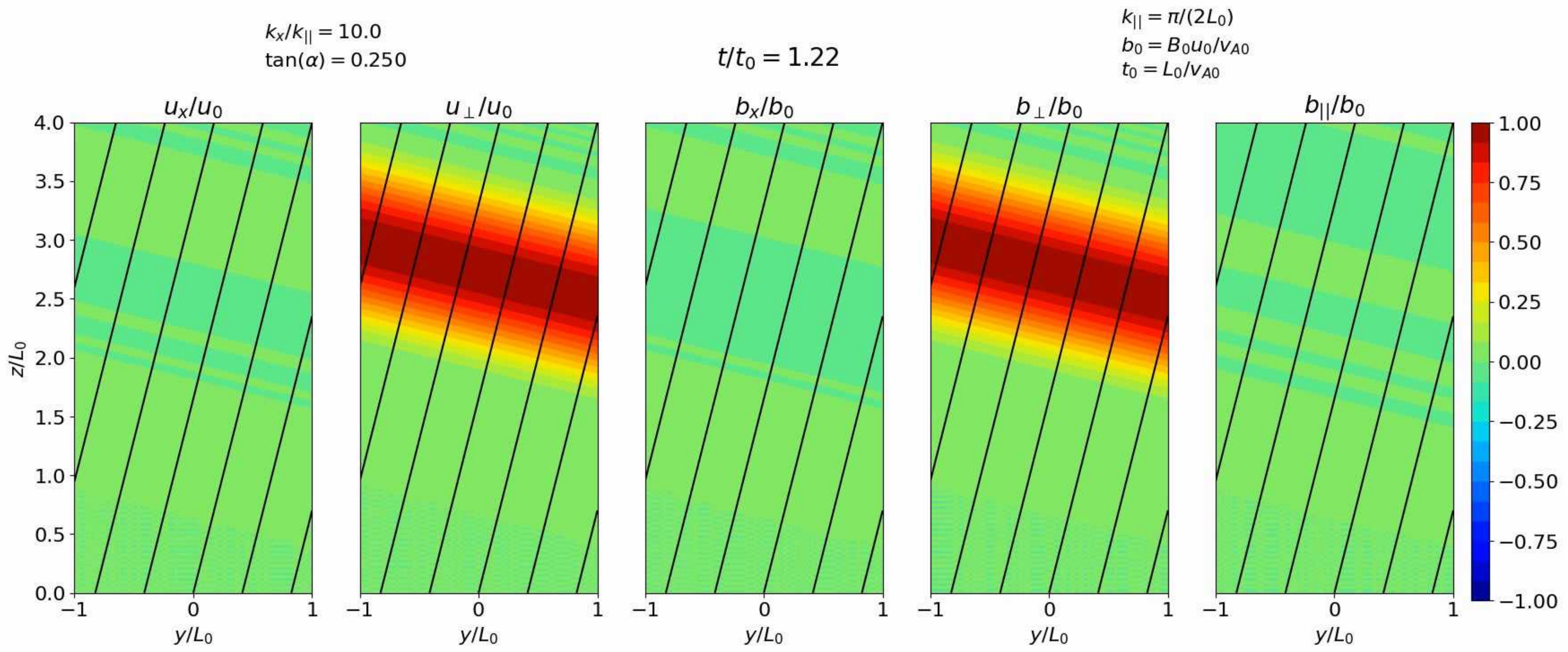
$k_x/k_{||} = 1.0$
$\tan(\alpha) = 0.000$
$t/t_0 = 1.22$
$k_{||} = \pi/(2L_0)$
$b_0 = B_0 u_0/v_{A0}$
$t_0 = L_0/v_{A0}$
u_x/u_0
$u_\perp/u_0$
b_x/b_0
$b_\perp/b_0$
$b_{||}/b_0$
z/L_0
y/L_0



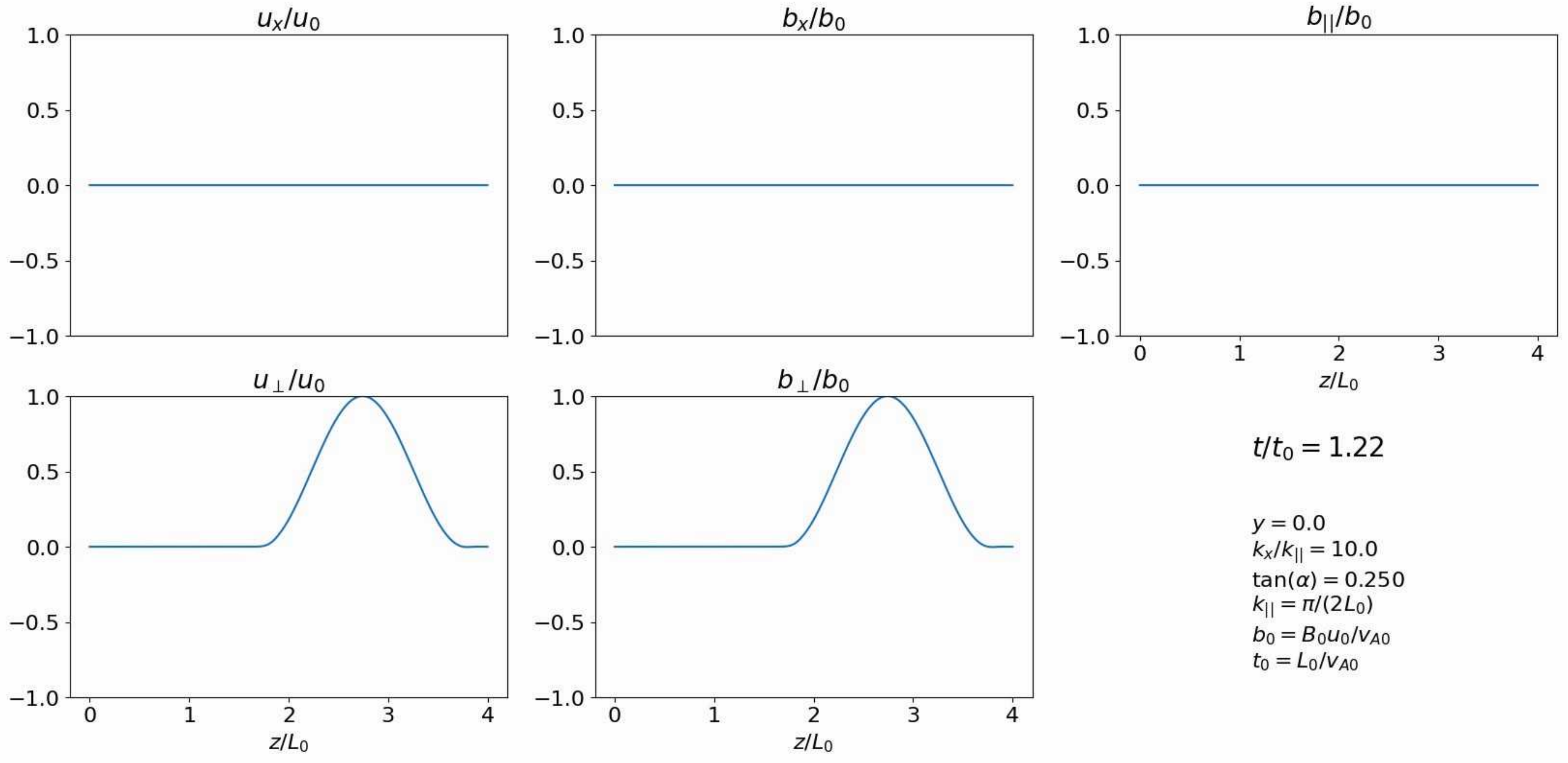
$k_x/k_{||} = 1.0$
$\tan(\alpha) = 0.250$
$t/t_0 = 1.22$
$k_{||} = \pi/(2L_0)$
$b_0 = B_0 u_0/v_{A0}$
$t_0 = L_0/v_{A0}$
u_x/u_0
$u_\perp/u_0$
b_x/b_0
$b_\perp/b_0$
$b_{||}/b_0$
z/L_0
y/L_0

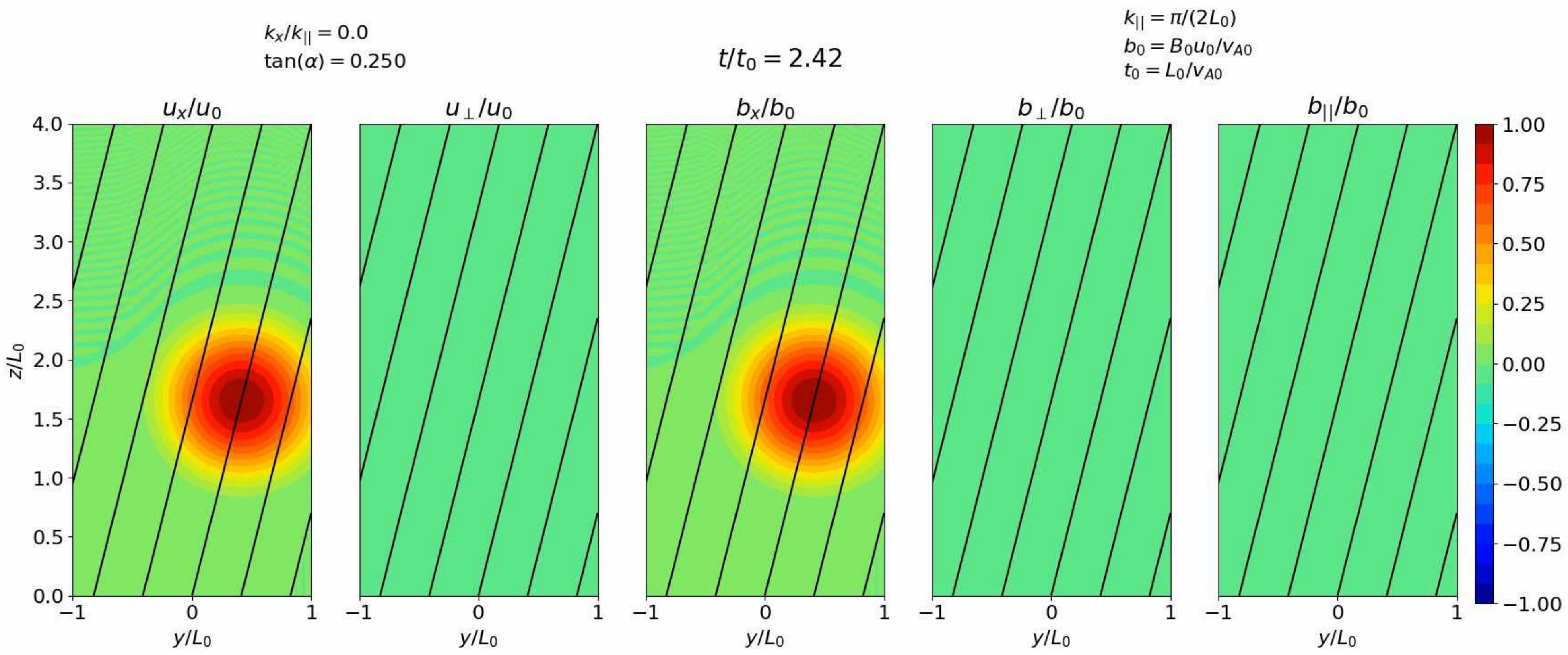


u_x/u_0
b_x/b_0
$b_{||}/b_0$
$u_\perp/u_0$
$b_\perp/b_0$
1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
4
z/L_0
$t/t_0 = 1.22$
$y = 0.0$
$k_x/k_{||} = 1.0$
$\tan(\alpha) = 0.250$
$k_{||} = \pi/(2L_0)$
$b_0 = B_0 u_0/v_{A0}$
$t_0 = L_0/v_{A0}$



$k_x/k_{||} = 10.0$
$\tan(\alpha) = 0.250$
$t/t_0 = 1.22$
$k_{||} = \pi/(2L_0)$
$b_0 = B_0 u_0/v_{A0}$
$t_0 = L_0/v_{A0}$
u_x/u_0
$u_\perp/u_0$
b_x/b_0
$b_\perp/b_0$
$b_{||}/b_0$
z/L_0
y/L_0

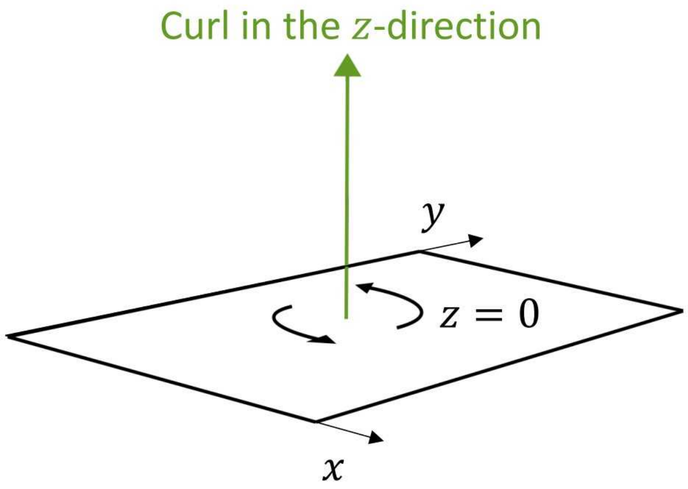

u_x/u_0
b_x/b_0
$b_{||}/b_0$
$u_\perp/u_0$
$b_\perp/b_0$
z/L_0
$t/t_0 = 1.22$
$y = 0.0$
$k_x/k_{||} = 10.0$
$\tan(\alpha) = 0.250$
$k_{||} = \pi/(2L_0)$
$b_0 = B_0 u_0/v_{A0}$
$t_0 = L_0/v_{A0}$



$k_x/k_{||} = 0.0$
$\tan(\alpha) = 0.250$
$t/t_0 = 2.42$
$k_{||} = \pi/(2L_0)$
$b_0 = B_0 u_0/v_{A0}$
$t_0 = L_0/v_{A0}$
u_x/u_0
$u_\perp/u_0$
b_x/b_0
$b_\perp/b_0$
$b_{||}/b_0$
z/L_0
y/L_0

Why does the coupling occur?

- $\frac{\partial b_z}{\partial t} = \hat{\boldsymbol{z}} \cdot \nabla \times (\boldsymbol{u} \times \boldsymbol{B_0})$
- $b_z = 0$ at $z = 0$
- $b_z = \cos(\alpha) b_{\parallel} - \sin(\alpha) b_{\perp}$
- $b_{\parallel} = \tan(\alpha) b_{\perp}$ $\Rightarrow$ Fast waves



Summary

At the solar surface:

- Alfvén waves couple to fast waves
- Change polarisation
- If

$$k_x^2 > k_{\|}^2 - k_y^2$$

then evanescent boundary layers form

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Normal mode solution

- Assume

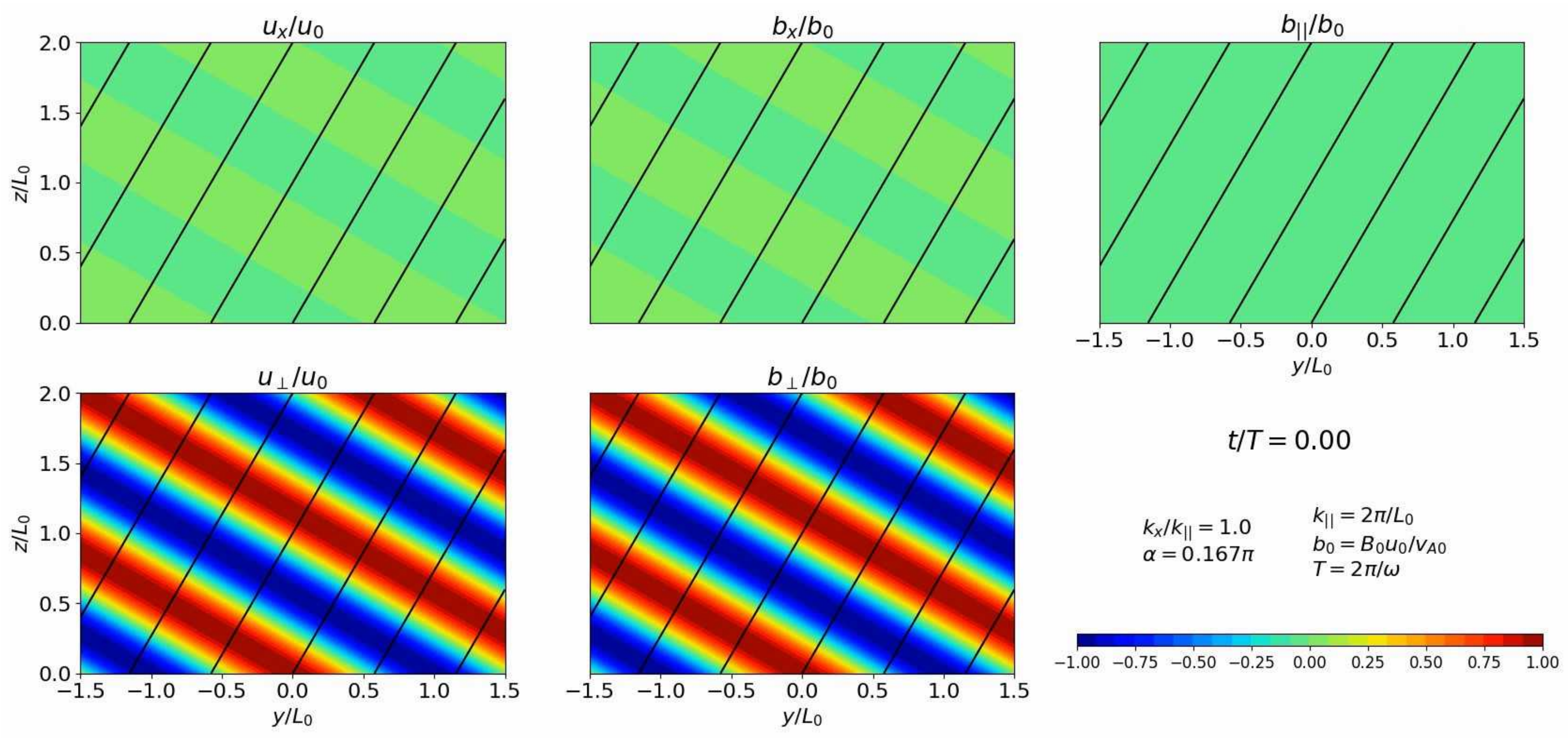
$$u_x,\ u_\perp,\ b_x,\ b_\perp,\ b_\| \propto \exp[i(k_x x + k_y y + \omega t)]$$

- Impose incident Alfvén wave

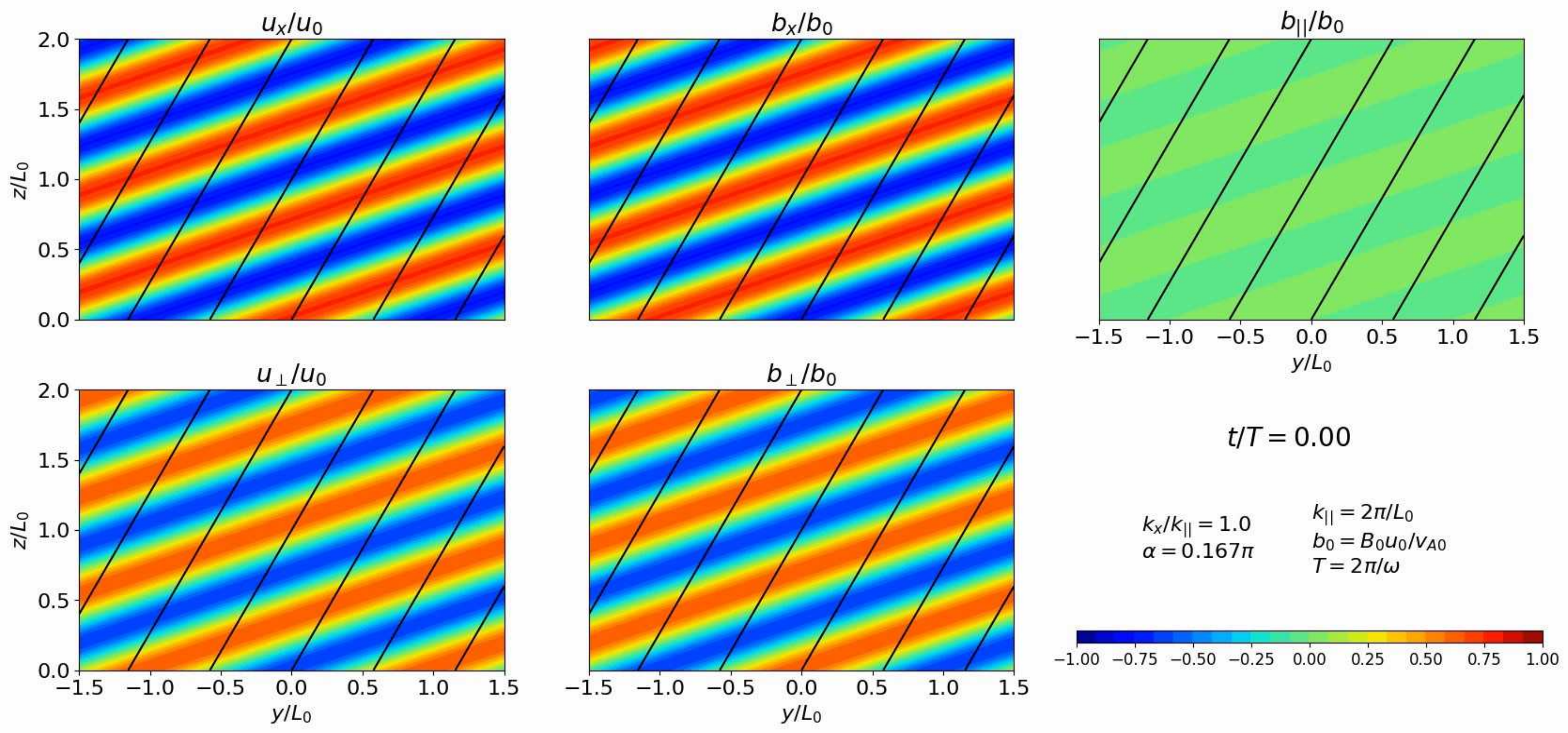
$$\frac{u_\perp}{u_0} = \frac{b_\perp}{b_0} = \exp[i(k_x x + k_\| s) + \omega t]$$

- Calculate unique reflected Alfvén and fast wave which ensures ***u* = 0**

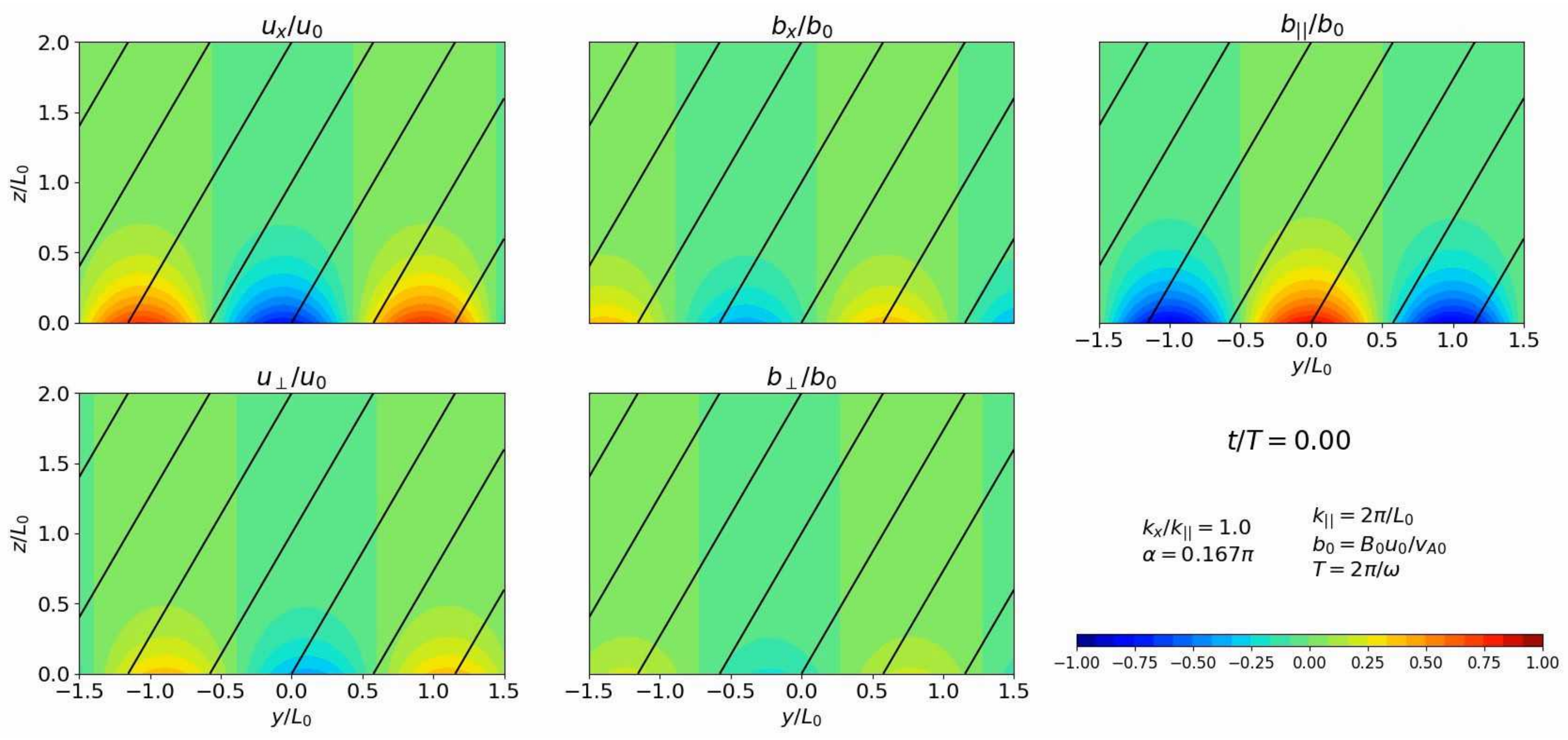
Incident wave



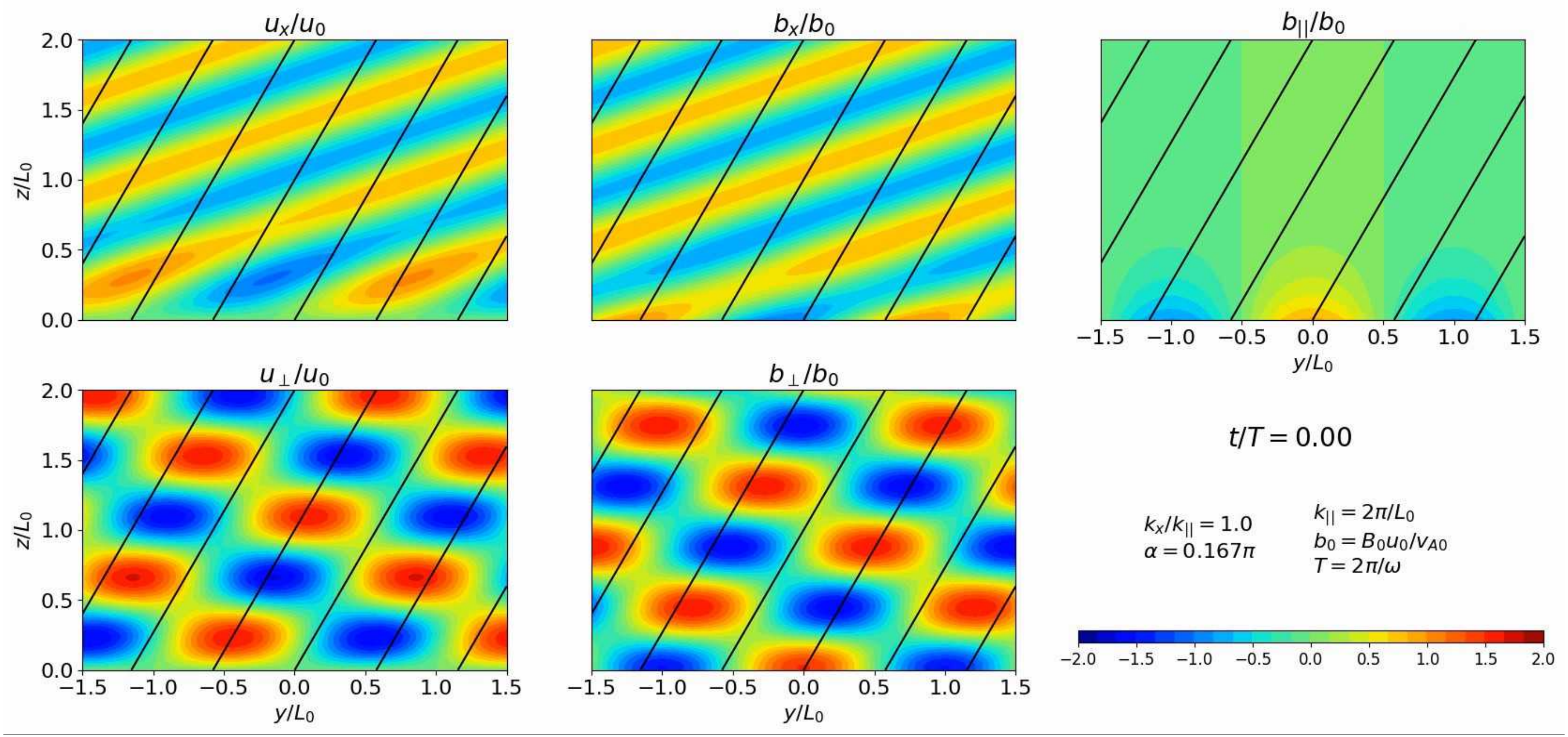
Reflected Alfvén wave



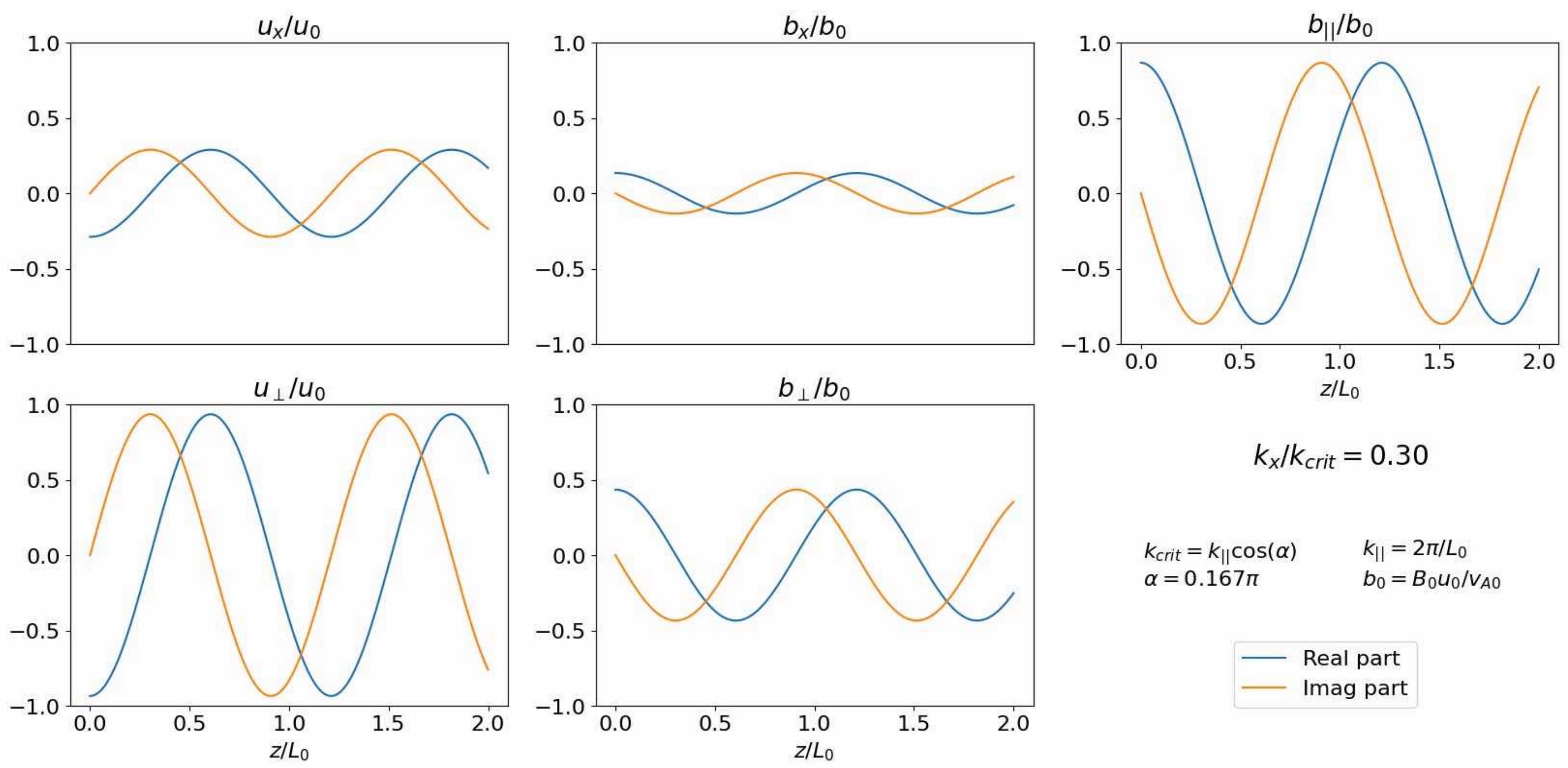
Reflected Fast wave



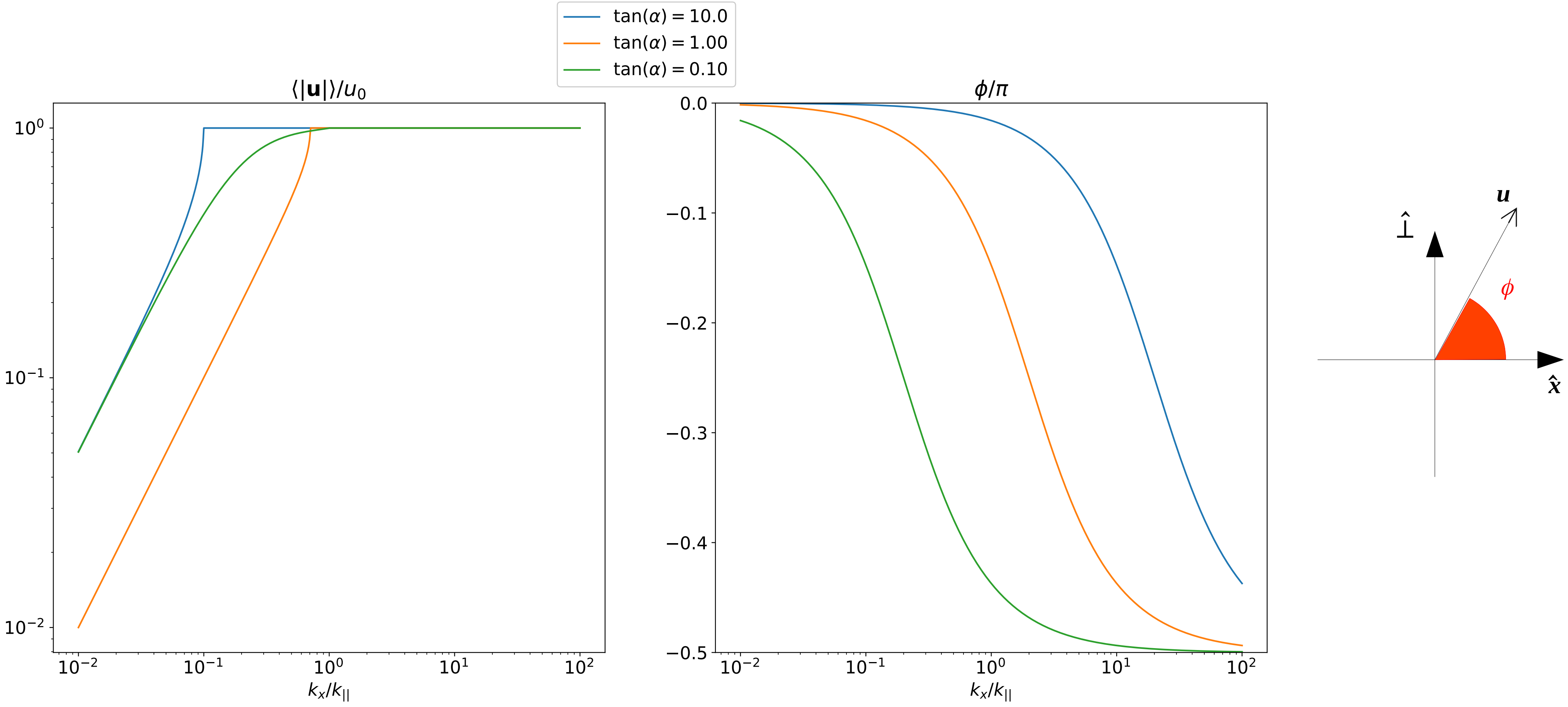
Full solution



Reflected fast wave



Reflected Alfvén wave



Summary

- Fast wave energy $\rightarrow$ 0 as $k_x \rightarrow \infty$
- Change in polarisation $\rightarrow$ 0 as $k_x \rightarrow \infty$
- $\therefore$ Boundary layers have a minimal impact on resonance absorption

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Model

- Background Alfvén speed:

$$v_A = \begin{cases} v_{A+} & \text{if} \quad z \geq 0 \\ v_{A-} & \text{if} \quad z < 0 \end{cases}$$

← Corona

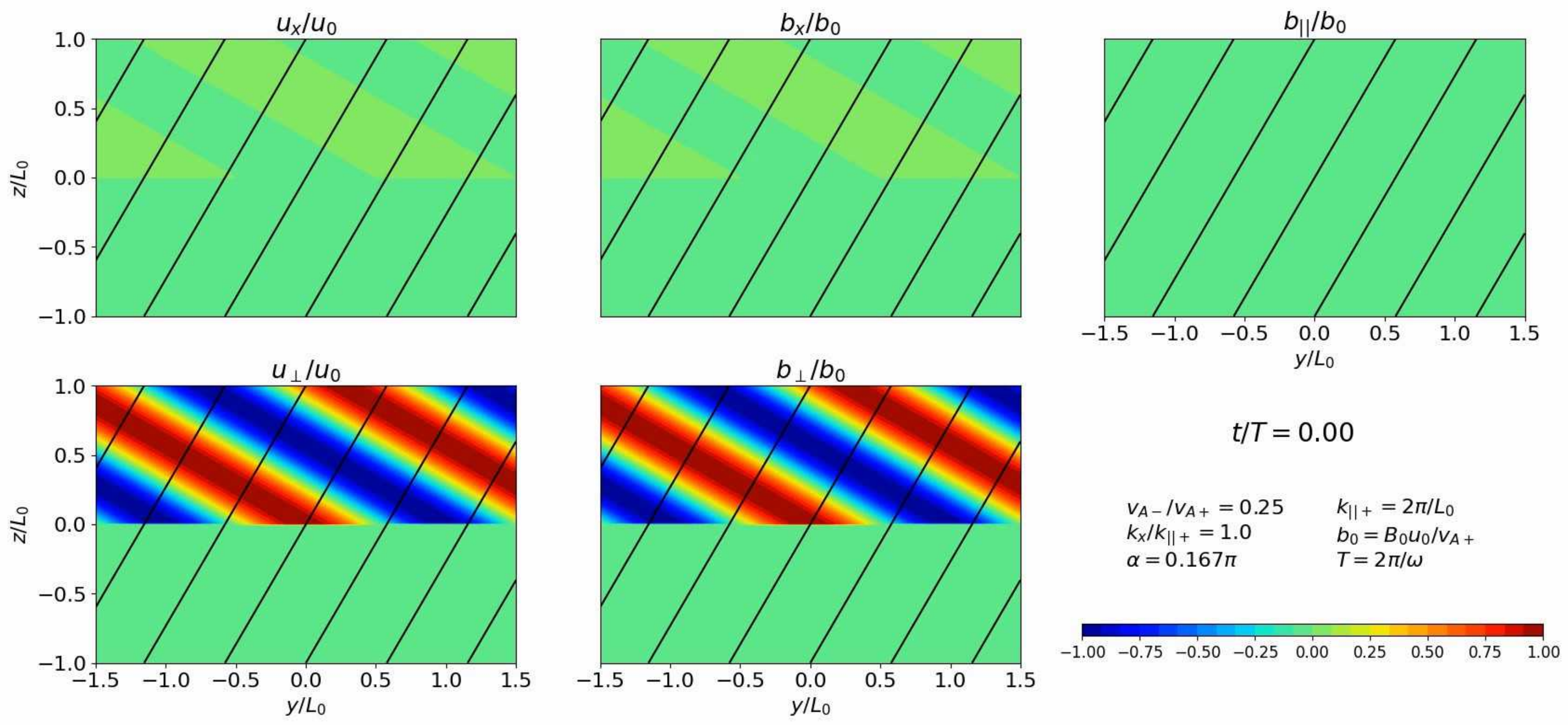
← Chromosphere

- Where:

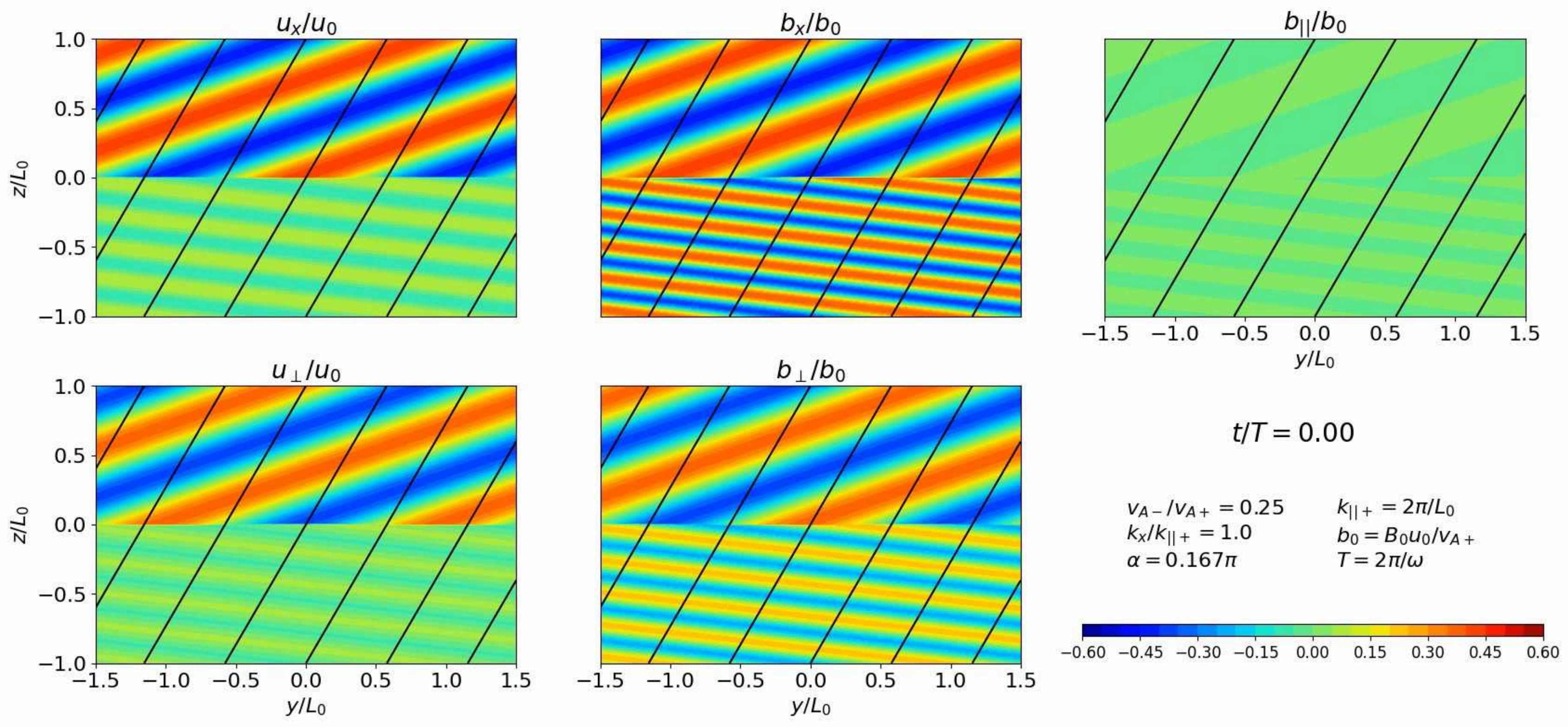
$$v_{A+} \gg v_{A-} \Rightarrow k_{\parallel -} \gg k_{\parallel +}$$

- Impose continuity of ***u*** and ***b***

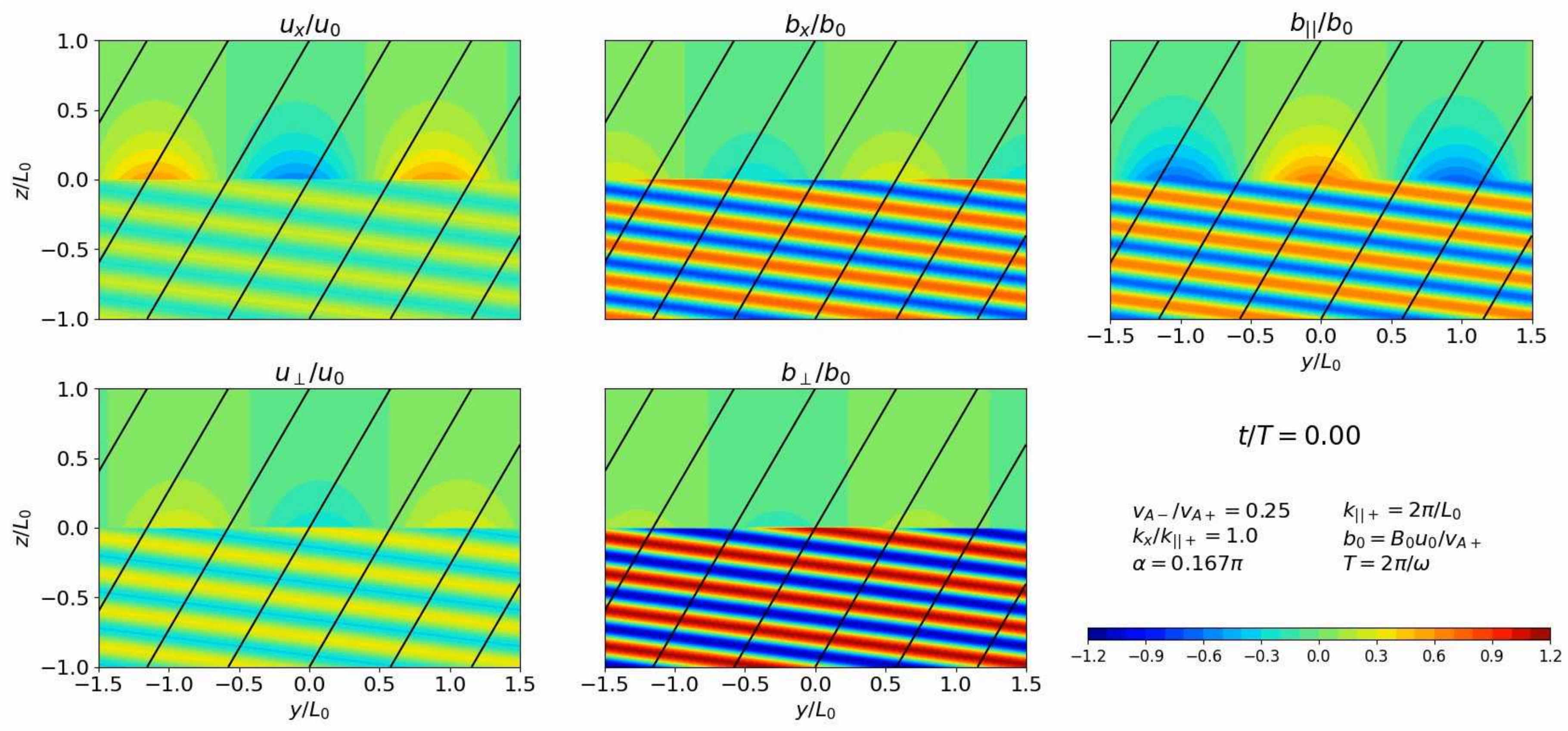
Incident wave



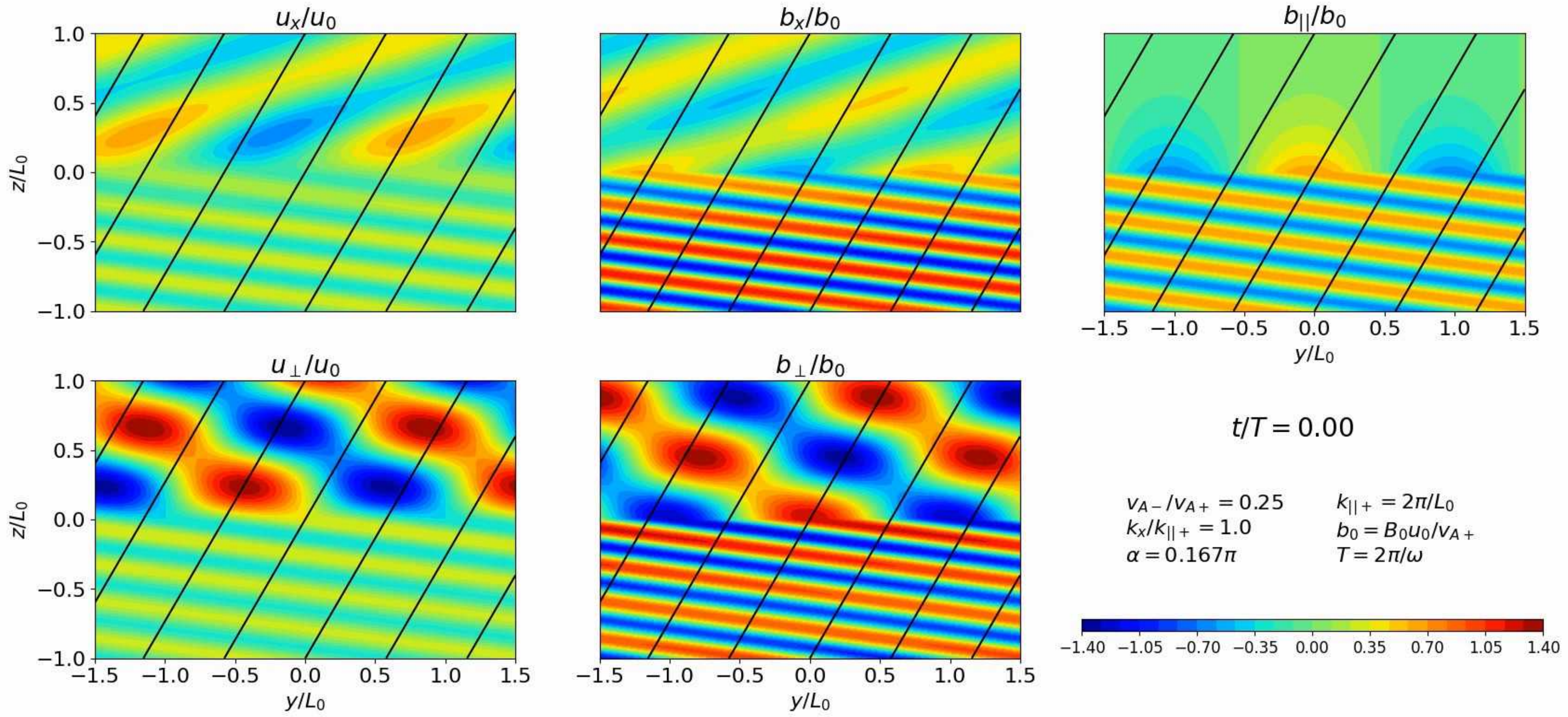
Reflected + Transmitted Alfvén wave



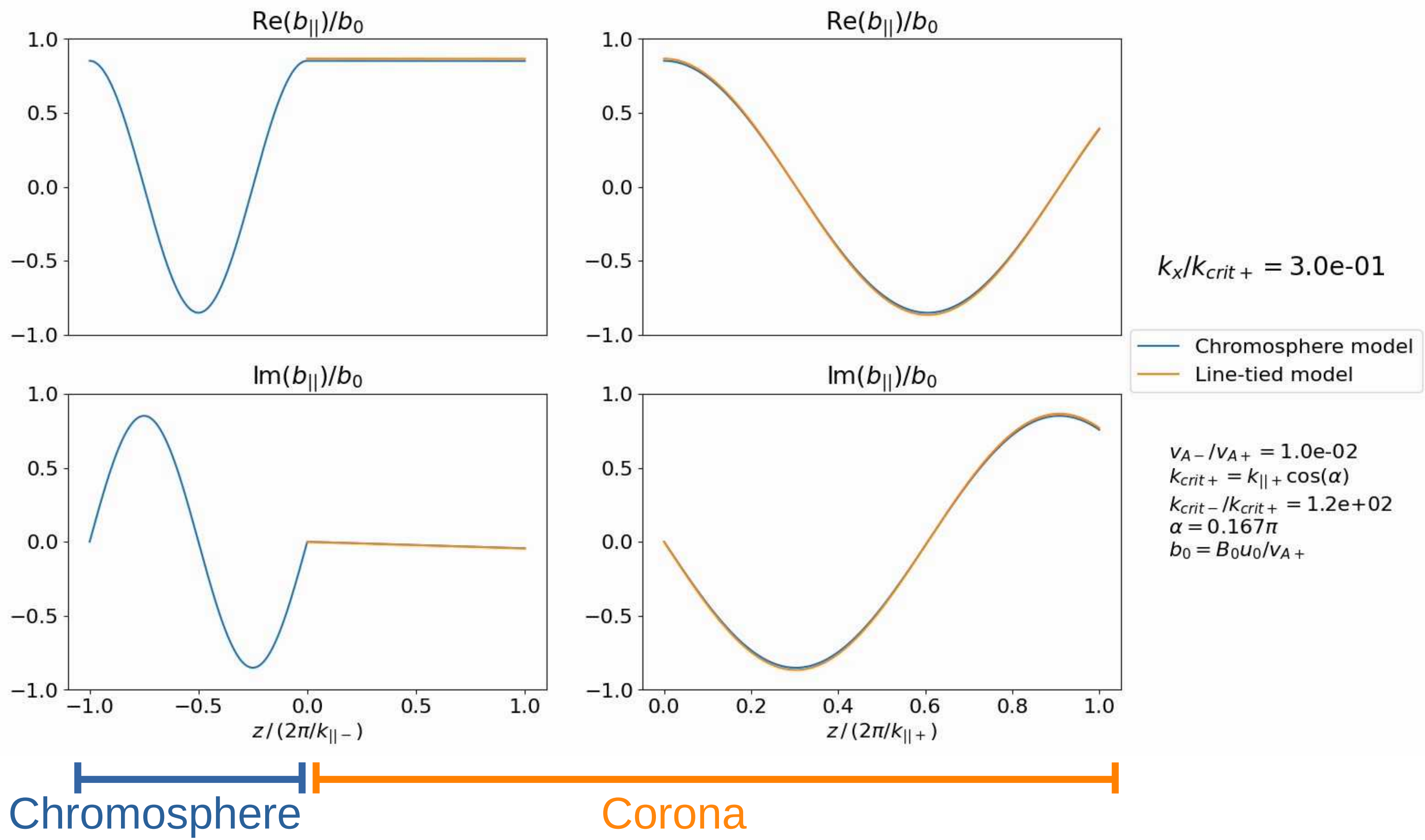
Reflected + Transmitted Fast wave



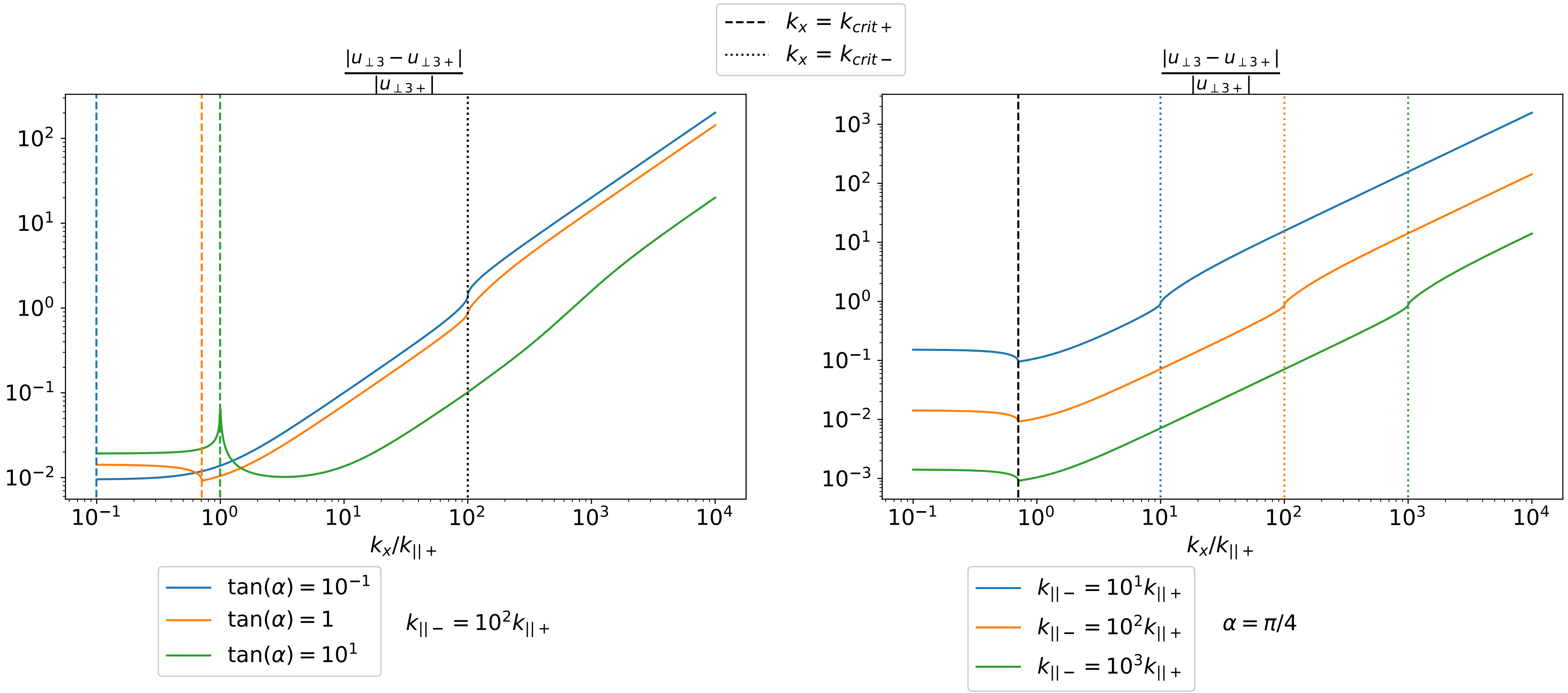
Full solution



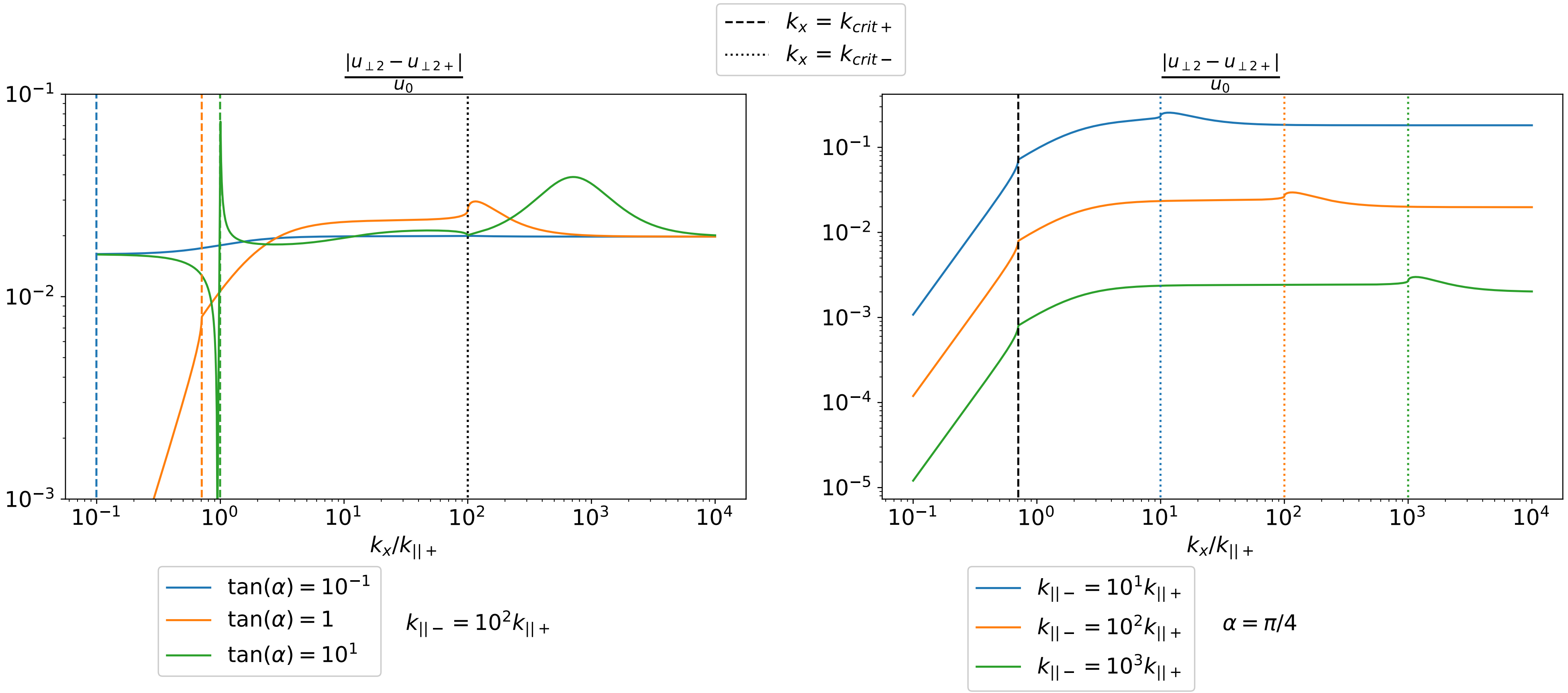
Chromosphere vs. Line-tied model



Reflected fast wave error



Reflected Alfvén wave error



Summary + conclusions

- Alfvén waves couple to fast waves at the TR
- They change polarisation upon reflection
- Line-tied BC's are usually a good approximation
- However, they generate unphysically large BL's if:

$$k_x \gg k_{\|-}$$

Future work

- Investigate different incident polarisations
- Use incident fast waves
- Model an exponential density profile instead of piecewise constant
- Let $\rho = \rho(x, z)$

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